Real Math Standards

for Kansan Public Schools

and other schools that desire better standards

so more children learn the language of numbers.

We begin with an overview of four Major Segments of mathematical instruction with increasing levels of specialty:

Early Primary: We master greater than and less than. We practice four basic operations of arithmetic – addition, subtraction, multiplication, and division – mostly with positive integers. We learn place values for ones, tens, hundreds, and thousands. We also learn place values for tenths and hundredths. We learn to count integers in both a positive direction and in a negative direction. We introduce negative integers as we move from Early Primary to Primary. We introduce positive fractions with integers in both the numerator and denominator and we learn to count by unit fractions that have a numerator of one and a positive single-digit denominator. We learn to count on the Real Number Line by both positive integer values up to 200 and by negative integer values from -1 to -100, as well as to count by basic unit fractions. We use no calculating machines, even with special education.*

Primary: We master arithmetic. Arithmetic includes the operations of addition, subtraction, multiplication, and division, as well as exponentials that raise integers to powers that are themselves integers. We learn to add and subtract both positive and negative fractions. We learn equivalent values for fractions and integers expressed as decimals and as percentages. We solve word problems with a variety of techniques. We practice arithmetic without the aid of a calculator. We develop knowledge and perspective of the language of numbers so that unknowns can be solved for, arithmetically or algebraically. We introduce functions with inputs and outputs. We introduce using a variable to represent a static quantity. We commit the multiplication table to memory. We practice fractions extensively.

Intermediate: We master the idea of equivalence. We master equivalent percentages, decimals, and fractions. We use algebra to solve for unknown quantities. We apply mathematics to word problems in order to solve for an unknown quantity or for an unknown relation, as in a function. We integrate technology into our computations after mastering basics of arithmetic and algebra. We prepare for a high school curriculum that may or may not include advanced algebra, statistics, trigonometry, or calculus. Not every student will be required to matriculate through Algebra II (or College Algebra). But

every* student will be able to substitute a variable to represent some unknown quantity with basic algebra. We also develop geometric reasoning during these intermediate "grades."

High School: We at Real Math Standards intentionally stopped short of writing standards for high school mathematics. The language of math itself is the better arbiter of how it should be proffered to different people as the cumulative nature of the language becomes apparent in secondary school. Mathematics is, of course, very sequential. Not every student should be expected to master conic sections, matrix manipulation, statistics, or plane trigonometry. Certainly the calculus is not for everyone. Not every student needs to calculate the slanted asymptote of a hyperbola in Cartesian Coordinates or find the zeroes of a fourth-order polynomial in one variable. Consequently, practice in the everyday mathematics that citizens regularly encounter will be the emphasis for a great many students in high school. But every* student will master basic algebra and, before that, arithmetic.

Our recommendation for high school mathematics is to have higher education (colleges and universities) develop their programs over time and to have our secondary schools (specifically, high schools) prepare students for college work in advanced algebra, trigonometry, statistical analyses, and classical calculus. A greater emphasis on Linear Algebra is the recommendation of our group, and that greater emphasis can actually begin in Primary school for students who do well with the language. For those students who are not four-year-college bound, or for those students who struggle with the language for any of a thousand different reasons, we insist that mathematics and its study in high school be both relevant and practical with an emphasis on arithmetic computation and basic algebraic relations. Spreadsheets should not be foreign or unknown to any of our students graduating high school. The well-established subjects of statistics and probability, plane trigonometry, and basic calculus, it is worth noting, need no re-write whatsoever from our group at Real Math Standards. The language is well established.

*Of course, as we are committed to help every student find success, we realize that certain students may never master basic algebra or even the basic facts of multiplication. Whether your inclination is to speak of "God-given ability" or of "natural ability" for our students, we realize that some very special people may never learn to read well or even memorize alphabetical order, let alone appreciate matrix manipulation or Cramer's Rule for solving simultaneous linear equations. We "never say never" with calculators, even in Early Primary math work. For example, some very special children on the autism spectrum often actually do better with inanimate hand-held tablets and other electronic devices than they do with live instruction. We are in our infancy in understanding these dynamics. Are these autistic students drawn to the "hand-held" nature of electronic devices, or their non-human nature? Is the constancy or reliability of the tablet the key? We do not yet know. Nevertheless, Real Math Standards are written with the tacit assumption that our academic standards are for the 99+% of students with general numerical ability and without "special" allowances such as those typically included within an IEP (Individualized Education Plan). We wholeheartedly recognize that some students lack the ability to handle even the most rudimentary arithmetic tasks or numerical analysis. Indeed, some children are very special. Our standards are written with the recognition that *virtually every student* can learn to multiply positive integers less than 16 and recall those basic facts, and combine those facts with others embedded in the operations of addition and subtraction of integers. To recognize equivalent fractions,

which is a necessary skill, both positive and negative integer values must be mastered with place values. All students need to understand the language of numbers. *Virtually every student* can master fractions.

Kindergarten

Recognize larger and small sizes of objects. Recognize larger and smaller sizes of sets (collections of like objects).

Count from 1 to 100 by ones, fives, and tens.

Count backward and forward from any integer one to 15.

Read digital and analog clocks. Read whole and half hours on analog clocks.

Know that calendars measure time.

Learn 24 hours equal one day. Learn 60 minutes equal one hour.

Learn concepts of morning, afternoon, evening, today, yesterday, tomorrow, hour, day, week month, and year.

Understand comparison words: more, more than, greater, greater than, less, less than, lesser, equal, equal to, equivalent, equivalence, same, some, many, all , few, none, most, and least.

Group 20 items into sets of 2, 4, 5, and 10.

Group 24 items into sets of 2, 3, 4, 6, 8, and 12.

Group 30 items into sets of 2, 3, 5, 6, 10, and 15.

Understand the number of objects is the same regardless of their arrangement or the order in which they were counted.

Count objects up to 20 arranged in various and/or scattered configurations.

Create rules and give appropriate rules for repeating and growing patterns with both numbers and shapes.

Model shapes from the real world by composing shapes from objects, e.g., sticks, clay balls, paper cut into familiar geometric shapes, etc.

Identify basic shapes: squares, circles, triangles, rectangles, pentagons and hexagons regardless of size or position. Count sides and vertices (corners).

Compose simple geometric shapes to form larger shapes, e.g., create a rectangle from two similar triangles.

Describe and/or compare locations of objects: inside, outside, between, above, below, over, under, near, far, up, down, in front (of), behind, top, bottom, top-middle-bottom, left-middle-right.

Identify coins by name and by value.

Sort and classify objects by color, shape, size, number, and other properties or characteristics.

Identify objects that do not belong to a particular set because of properties or attributes.

Grade 1

Read digital and analog clocks.

Name and count days of the week.

Name and count months of the year.

Know ages (in years) of classmates and siblings.

Understand that a square is a rectangle but a rectangle is not (necessarily) a square.

Mentally determine 10 more or 10 less than a given two-digit number without having to count.

Count even and odd integers by twos.

Add and subtract integers up to 50.

Recognize subtraction as a difference.

Determine whether simple equations are true or false. Change a “false equality” to an inequality with > or < to make a statement true.

Learn three-dimensional shapes: cubes, cones, cylinders, and spheres. Understand relationships of each to two-dimensional triangles, squares, circles, and rectangles.

Draw (create) two-dimensional shapes with defining properties.

Use number lines, bar graphs, and line graphs.

Measure using an inch ruler; measure to nearest inch, half-inch and quarter-inch.

Measure using a metric ruler; know ten millimeters equal one centimeter.

Read a thermometer to integer values and to half-degrees.

Find the value of sets of pennies, nickels, dimes and quarters in various combinations.

Add three integer-value addends.

Compare unit fractions: halves, thirds, fourths, and fifths. Determine which is bigger from “sliced” objects as well as from pictures as examples.

Divide shapes into equal parts.

Learn ordinal numbers: first, second, third, etc., up to tenth.

Use direct comparison as well as nonstandard units to compare and order objects and ideas (such as time) according to length, area, volume (capacity), weight, duration, and temperature.

Understand place value for one, tens, and hundreds.

Add a one-digit number to a two-digit number with regrouping (carrying).

Organize and interpret data with up to four choices or data points.

Grade 2

Read and write integers up to one thousand.

Add four or more addends of both one- and two-digit positive integers.

Lean simple ideas of probability: impossible, unlikely, probable (or likely), and certain.

Memorize multiplication tables through $10 \times 10 = 100$.

Count money and compare monetary values up to $5; make change; decide if an amount is enough to make a purchase.

Identify and compare fractions with like numerators and/or like denominators.

Engage with word problems having integer values up to 100 using addition, multiplication, subtraction, and division.

Estimate by tens and by hundreds before embarking on challenging problems with values greater than 250 (or so).

Learn units of length: inch, foot, yard, centimeter, millimeter, meter.

Learn units of time: second, minute, hour, day, year.

Learn units of volume and capacity: pint, quart, gallon, cm^3, liter, $in.^3$, $ft.^3$, $yd.^3$.

Introduce the Associative Property; introduce the Distributive Property.

Introduce the Commutative Properties of addition and multiplication.

Recognize that Associative, Commutative, and Distributive Properties are innate and “have been with us” since we started learning about the language of numbers.

Use place-value understanding to compare two three-digit integers based on value of digits and their meaning with place value; use >, =, and < symbols to record results of those comparisons.

Grade 3

Read and write positive integer values up to 10,000. Use words, models, standard form and expanded form to represent and show equivalent forms of positive integers with three and four digits.

Compare two positive integers up to 10,000 using >, =, and < symbols.

Round positive integers with two-to-four digits to the nearest 10, 100, or 1000 (as appropriate).

Add and subtract positive integer values fluently within 1000.

Multiply and divide positive two-digit values where the dividend is a multiple of the divisor, i.e., the quotient is itself an integer.

Place basic unit fractions onto the Real Number Line. Those unit fractions should include ½, 1/3, ¼, 1/5, 1/6, 1/8, and 1/10.

Represent a fraction, 1/x, on a number line by defining the interval from 0 to 1 as the whole, and partitioning it into x equal parts. Recognize that each part has size 1/x and that the endpoint of the segment placed at (beginning at) zero locates the number 1/x on the number line.

Understand two fractions are equivalent (of equal value) if they are the same size, based on the same whole or the same point on a number line.

Recognize simple equivalent fractions, e.g., 1/2 = 2/4 = 5/10; 4/6 = 2/3 = 6/9. Show (demonstrate, explain, or simply recognize) why the fractions are equivalent (as by using a visual fraction model).

Recognize integer-value quotients. Demonstrate, explain, or simply recognize 63 ÷ 9 as the number of objects in each share when 63 objects are partitioned equally into 9 shares, with the integer quotient of 7 objects within each share.

Find fractions equivalent to integers.

Represent the operation of multiplication of integers with the following models: equal-sized groups, rectangular arrays and/or areas, and equal "jumps" on a number line.

Understand the zero property of multiplication (a times 0 equals zero) and the identity property of multiplication (a times 1 equals a).

Understand the commutative property of multiplication (a times b equals b times a).

Expand the commutative property of addition to more than two addends (a + b + c equals b + c + a equals c + b + a; 3 + 4 + 5 + 6 = 5 + 4 + 6 + 3, etc.)

Demonstrate fluency with memorization of multiplication facts and corresponding division facts through 15 × 15 = 225. Consequently, division of integers up to 200 (dividends) by integers up to 15 (divisors)

should result in quotients expressed both with remainders and with any remainder expressed as the numerator of a fraction with the original divisor as the denominator.

Know length equivalents: 12 inches equal one foot; 3 feet equal 36 inches equal one yard; 100 cm equal one meter; 1000 mm equal one meter; 1000 m = one kilometer.

Know time equivalents: 7 days equal one week; 24 hours equal one day; 60 minutes equal one hour; 60 seconds equal one minute.

Write times from analog clock to nearest minute. Determine elapsed times to nearest five minutes.

Know the difference between an equality and an inequality.

Compare decimals expressed to tenths or hundreds with >, =, <.

Add four or more decimal values.

Know even and odd integers.

Grade 4

Read and write positive integer values up to 100,000. Use words, models, standard form and expanded form to represent and show equivalent forms of positive integers with as many as seven digits.

Round positive integers less than 10,000,000 to any given place value.

Write tenths, hundredths, and thousandths as both decimals and as fractions. Use words, standard form and expanded form to represent decimal numbers to thousandths.

Add four or more decimal values.

Learn 2.54 cm equals one inch.

Solve real-world problems involving addition and subtraction of positive integer values up to 100,000. Employ equations with an unknown number as a symbol (a variable) to represent the problem.

Memorize the fraction and decimal equivalents for halves, thirds, and fourths, e.g., 1/2 = 0.5 = 0.50; 7/4 = 1 3/4 = 1.75; 4/6 = 2/3 = 0.666…

Compare two positive integers up to 1,000,000 using >, =, and < symbols.

Compare two decimal values expressed to nearest tenth or to nearest hundredth, both positive and negative, using >, =, and < symbols.

Multiply integer values fluently within 200.

Add and subtract mixed numbers with common denominators.

Add and subtract reduced fractions with different denominators.

Use commutative property of multiplication and associative property of multiplication to show that factors may be multiplied in any order. Use the distributive property to multiply across binomials.

Establish relationships between addition and multiplication, between subtraction and division, and the inverse relationship between multiplication and division to solve real-world and other mathematical problems. Write word problems that illustrate mathematical statements.

Interpret 42 as 6 × 7 then as a statement that 42 is 6 times as many as 7, and 7 times as many as 6. Represent verbal statements of multiplicative comparisons as multiplication equations. Establish that ab = (a/2)×(2b). Interpret 6 × 7 as 2 × 3 × 7 = 3 × 2 × 7 = 3 × 14 = 42.

Measure length with English units with both rulers and tape measures; measure to the nearest quarter-inch, eighth-inch, and sixteenth-inch. Measure length with metric units with both rulers and tape measures; measure to the nearest centimeter and nearest millimeter.

Learn (commit to memory) basic comparisons of both English and metric units. Know relative sizes of measurement units within both systems of units, including km, m, cm, mm; kg, g; lb., oz.; l, ml; hr., min., sec. Express measurements in a larger unit in terms of a smaller unit within a single system of measurement. Record measurement equivalents in a two-column table.

Measure angles in integer-value degrees using appropriate tools. Sketch angles of specified measure; know that angles are formed by two rays that share a common endpoint.

Employ area and perimeter formulas for triangles and rectangles to solve given word problems. Find the area of complex shapes composed of rectangles and/or triangles by decomposing them into non-overlapping rectangles and triangles then adding the areas of the non-overlapping parts.

Learn that an angle can be measured with reference to a circle, with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. Understand that 1/360 of a revolution (of a circle) is a one-degree angle, written 1° and can be used to measure other angles. Establish the angle addition postulate.

Know that an n-degree angle is equivalent to one that rotates through n one-degree angles.

Use the four operations or arithmetic (addition, subtraction, multiplication and division) to solve given real-world word problems involving distances, intervals of time, volumes, and money. Include addition and subtraction problems involving simple fractions and problems that require measurements expressed in one unit in terms of a different unit for the same physical entity.

Formulate questions that can be answered with data. Use observations, surveys, and experiments to collect, represent, and interpret the data using tables, including frequency tables or histograms, line plots, pie charts, and bar graphs.

Establish mode, median, range, and mean for simple data sets.

Display a data set of measurements in unit fractions (1/2, 1/4, 1/8). Solve problems with addition and subtraction of fractions using data displayed in line plots.

Establish concepts of parallel and perpendicular. Categorize (classify) triangles and quadrilaterals based on the presence or absence of parallel or perpendicular lines (or line segments). Establish that congruent means “of the same measure.”

Establish that a plane is determined by two non-skew lines.

Establish that a point has no size; it is a mathematical idea, a location in three-dimensional space.

Establish dimensions and illustrate each with real-world examples, e.g., the movement of planes, trains, and automobiles might be described as three-dimensional, one-dimensional, and two-dimensional, respectively.

Construct parallelograms, rhombuses, rectangles, and trapezoids with tools. Employ ruler, straightedge and/or computer applications to construct a variety of plane (two-dimensional) figures.

Compare integer values up to ten billion with >, =, <.

Practice estimation in a variety of ways, including rounding and reduction of number of significant digits.

Divide two-digit integers by one-digit integers from both math statements and word problems.

Divide larger integers by one-digit integers and interpret remainders.

Divide three- and four-digit integers by two-digit integers.

Calculate volumes of orthogonal parallelepipeds (shoeboxes and rectangular aquariums).

Compare and order both positive fractions, mixed numbers, and decimals to thousandths and ten thousandths using >, =, and < symbols.

Grade 5

Round decimal numbers from thousandths to thousands to any prescribed place value.

Practice percentages as a fraction of one hundred with drawings, tables, diagrams, and other visual models.

Compare and order both positive and negative fractions, mixed numbers, and decimals to thousandths and ten thousandths using >, =, and < symbols.

Know a digit in a multi-digit number represents 10 times as much as it would represent in the place to its right, and inversely, a digit represents 1/10 of what it would represent in the place to its left.

Establish patterns in the number of zeros and the placement of the decimal point when multiplying a number by powers of 10; employ positive and negative integer exponents to denote powers of 10.

Add and subtract fractions, including mixed numbers, with differing denominators.

Evaluate expressions when there are no parentheses to specify an order of operations, e.g., use the formula $V = s^3$ to find the volume of a cube or $A = 6s^2$ to find the surface area of a cube with sides given as an integer, fraction, mixed number, or decimal value.

Convert time units from two- and three-digit integer hours to days-and-hours.

Convert time units from three- and four-digit integer minutes to (days)-and hours-and-minutes.

Practice integer divisibility rules for 2, 3, 4, 5, 6, 8, 9, and 10.

Divide fractions by integers.

Divide fractions by unit fractions.

Divide fractions and mixed numbers.

Divide two fractions.

Identify types of triangles: acute, right, obtuse, scalene, isosceles, equilateral.

Classify quadrilaterals: parallelograms, trapezoids, rhombuses, kites, rectangles, squares.

Understand similar and congruent.

Learn perimeter and formulas for: rhombus, rectangle, square, triangle.

Learn area and formulas for: triangle, rectangle, square, circle.

Learn pi as 3.14159. Understand pi as a decimal is an approximation.

Grade 6

Relate both positive and negative real numbers (integers and decimal values) to real-world quantities.

Place positive and negative numbers (integers and decimal values) on the Real Number Line.

Relate Absolute Value to distance on the Real Number Line.

Know difference between prime and composite numbers.

Understand Greatest Common Factor and Least Common Multiple

Relate ratios and unit rates to real-world quantities.

Divide both integers and decimal values without a calculator using one-digit and two-digit divisors.

Develop fluency with Order of Operations with parentheses, exponents, and the four basic operations of arithmetic.

Evaluate variable expressions with specific values.

Employ variable expressions to represent dynamic real-world quantities.

Compare algebraic statements to determine if they are equivalent.

Graph ordered pairs in Cartesian Coordinates.

Graph two-variable data to analyze the relationship between the variables.

Graph solutions to inequalities on the Real Number Line.

Convert between English and metric units.

Know properties of triangles and quadrilaterals.

Measure distance between points on a coordinate plane.

Calculate volumes of basic shapes.

Determine areas of complex shapes with decomposition into simpler shapes.

Understand that statistical distributions have spread and shape.

Practice collecting real-world data.

Select and implement graphical representations to display real data.

Evaluate trends in dynamic data.

Grade 7

Find Prime Factorization of positive integers.

Add and subtract distances on the Real Number Line.

Understand Absolute Value of variables.

Compute unit rates and convert units of comparable physical entities.

Solve ratio and percent problems with real-world applications, e.g., tax, discounts, interest.

Simplify expressions with laws and properties (identity, commutative, associative, distributive, etc.)

Understand slope a vertical change per unit of horizontal change.

Define a linear function of the form $y = mx + b$.

Graph a line in Cartesian Coordinates given a slope and a point on the line.

Find the slope of a line from a graph.

Solve problems of adjacent angles, complementary angles, and supplementary angles.

Use similar triangles and ratios to solve problems; include word problems in that work.

Know acute triangles, obtuse triangles, scalene triangles, right triangles, isosceles triangles.

Know equilateral triangles are equiangular.

Understand the difference between a statistical sample and a population.

Calculate volumes and surface areas of simple three-dimensional objects.

Know formulas for area of a circle and circumference of a circle. Know pi to five digits (3.14159).

Understand that all probabilities are represented between 0 and 1 on the Real Number Line.

Understand that zero probability is “impossible” and probability of 1 is “certain.”

Learn sample spaces and probabilities of events with equally likely outcomes.

Grade 8

Understand decimal representations of both rational and irrational numbers.

Use Scientific Notation to express real-world values.

Solve linear equations with rational coefficients.

Graph linear functions in Cartesian Coordinates.

Determine whether two linear equations have one solution, no solution, or infinite solutions.

Understand that the intersection of two lines in Cartesian Coordinates is a simultaneous solution.

Solve real-world problems with three-dimensional objects and their cross-sections.

Use the Pythagorean Theorem to find an unknown side of a right triangle.

Use the Pythagorean Theorem and/or Distance Formula to determine distance between two points.

Construct Scatterplots and describe patters, trends, and correlation between abscissa and ordinate.

Understand independent and dependent events.

Understand complementary and mutually exclusive events.